An Introduction to Solid State Concepts

PHYS 21103

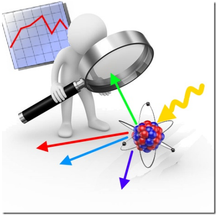
Department of Physics, Instructional Labs

University of Chicago

Overview

Solid State Experiments in PHYS 211

- Electrical Resistivity
- Specific Heat
- Hall Effect in Semiconductors
- Optical Absorption Edge of Semiconductors
- Mössbauer Spectroscopy of ^{57}Fe



A pretty accurate representation of how one studies solid state physics

[Source: *Solid State Physics Group*, Department of Physics, University of Torino]

Overview

Band Structure
- From energy levels to bands
- Conductors, insulators and semiconductors

Electrical Conduction
- Free electrons
- Drift velocity and current
- Ohm's law

Lattice Vibrations
- The Einstein solid
- The Debye model
- Phonons

Superconductivity

Band Structure

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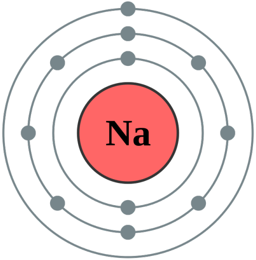
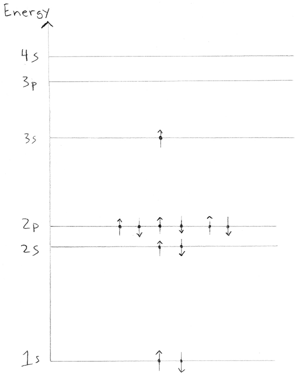
Band Structure: An Example

Sodium: Na (Z=11): $(1s)^2(2s)^2(2p)^6(3s)^1$



[Source: Wikimedia Commons]



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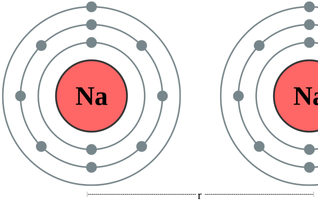

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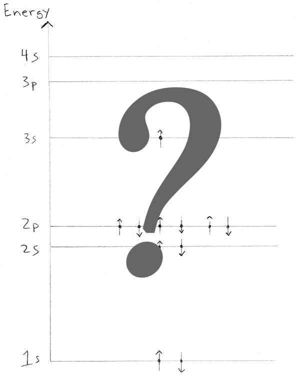

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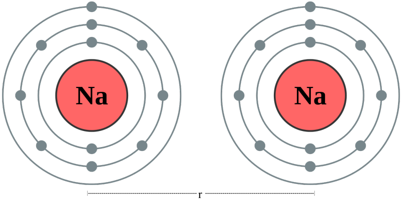
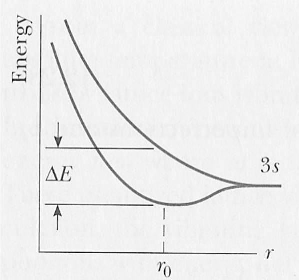


[Source: Wikimedia Commons]



2 atoms at distance *r*
[Source: Fig. 12.16, *Modern Physics*, Serway, Moses & Moyer, 2005]

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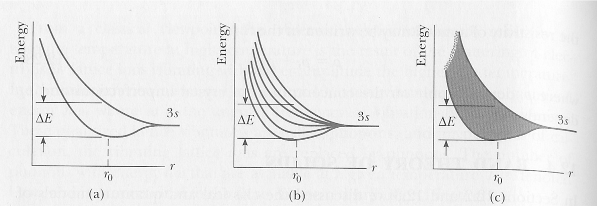
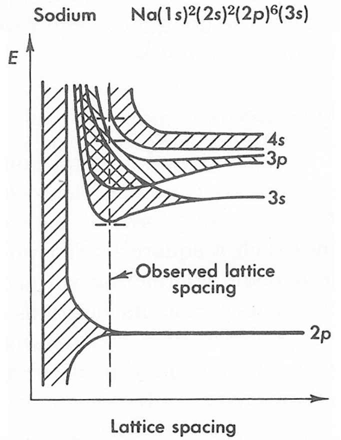


(a) 2 atoms, (b) 6 atoms, (c) many atoms
[Source: Fig. 12.16, *Modern Physics*, Serway, Moses & Moyer, 2005]

Band Structure: More Details



[Source: Fig. 3.7, *Experiments in Modern Physics*, Melissinos, 1966]

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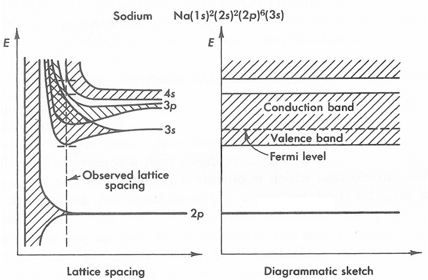
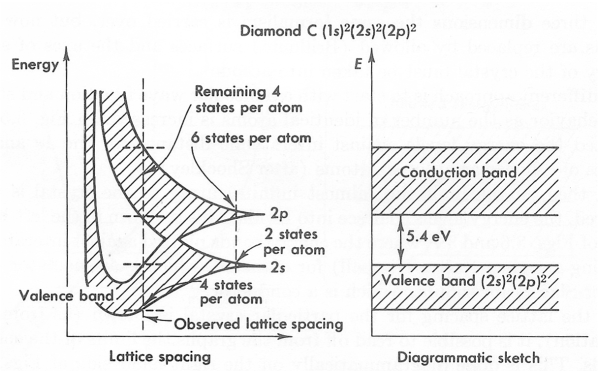


[Source: Fig. 3.7, *Experiments in Modern Physics*, Melissinos, 1966]

- Electrons fill the bands in the ground state up to the **Fermi Level**
- The highest band with electrons in the ground state is the **Valence Band**
- The lowest band with openings in the ground state is the **Conduction Band**

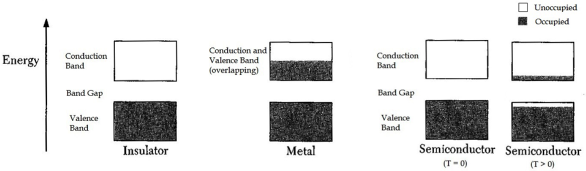
Band Structure: More Details



[Source: Fig. 3.6, *Experiments in Modern Physics*, Melissinos, 1966]

Note that in the ground state the valence band is **filled** and the conduction band is **empty**.

Band Structure: Conductors, Insulators and Semiconductors



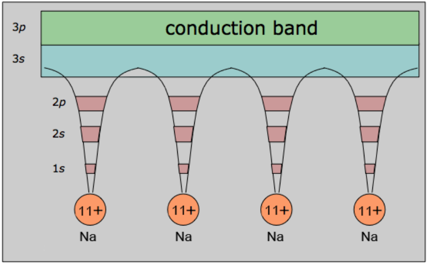
[Source: Fig. 9.1, *Introduction to Solid State Physics, 3rd Ed.*, Kittel, 1966]

- **INSULATORS**: the valence and conduction bands are separate with a large band gap (typically several eV or more)
- **CONDUCTORS**: the valence and conduction bands overlap
- **SEMICONDUCTORS**: the valence and conduction bands are separate in the ground state with a small band gap (typically 0.1-1 eV)

Electrical Conduction

Conduction: Introduction

- Electrons in the valence band cannot move far from their nucleus
  - they are **localized**
  - insulators have **no free electrons**
- Electrons in the conduction band are nearly free
  - they can move about the crystal
  - it costs very little energy to excite electrons
  - free elections lead to **conduction**



[Source: *Bonding in Metals and Semiconductors*, http://chemwiki.ucdavis.edu/]

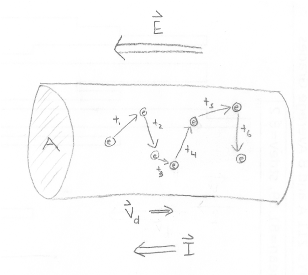
Conduction: Drift Velocity and Current

When an electric field is applied, electrons feel a force

$$\mathbf{F} = -e\mathbf{E}$$

They scatter with an average time τ between collisions and develop a **drift velocity**

$$\mathbf{v}_d = -e\mathbf{E}\tau/m$$



Therefore, we have a net current I and can define a current density,

$$\mathbf{j} = \mathbf{I}/A = -ne\mathbf{v_d} = ne^2\mathbf{E}\tau/m$$

where n is the number of free electrons per unit volume, and A is the cross sectional area of the material.

Conduction: Ohm's Law

Current density:

$$\mathbf{j} = \mathbf{I}/A = -ne\mathbf{v_d} = ne^2\mathbf{E}\tau/m$$

If we rearrange this, we find the fundamental form of **Ohm's law**,

$$\mathbf{j} = \sigma\mathbf{E} \text{ or } \mathbf{j} = \mathbf{E}/\rho$$

where

$$\sigma = ne^2\tau/m \text{ is the conductivity}$$

or

$$\rho = m/ne^2\tau \text{ is the resistivity}$$

Conduction: What causes scattering?

So far we have not mentioned what causes electrons to scatter.

It's not the ions!

- In a perfectly periodic crystal, there is no electron-ion scattering
- But electrons scatter off things which *break* the periodicity

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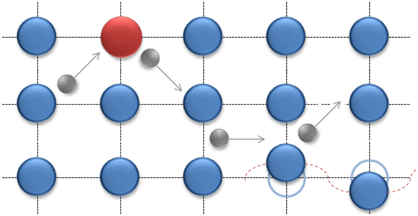
The two main contributions to resistivity (and which set the time scale τ) are:

- scattering off defects
- scattering from lattice vibrations (phonons)



[Source: Morelli Research Group, Michigan State University, egr.msu.edu/morelli-research]

Conduction: Where do we use this?

We will study conduction in:

- Electrical Resistivity
    - Resistivity of electrons in metals
    - Resistivity of electrons in semiconductors
- Hall Effect
    - Resistivity of electrons and holes in semiconductors
    - Mobility of electrons and holes in semiconductors
    - Magnetoresistance

Lattice Vibrations

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  From energy levels to bands
  Conductors, insulators and semiconductors

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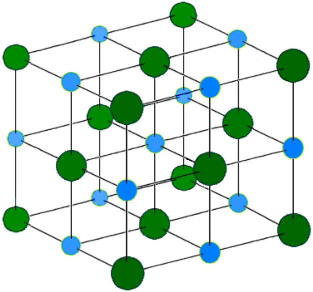
  The Einstein solid
  The Debye model
  Phonons

Superconductivity

Lattice Vibations: Introduction

The atoms in a solid arrange themselves in a periodic array known as a **lattice** or **crystal**.



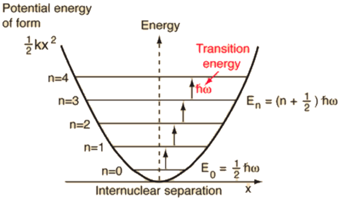
[Source: *Wikimedia Commons*]

However, adding energy to the atoms makes them **vibrate**.

Lattice Vibrations: Einstein Solid

Albert Einstein proposed that these atoms vibrate independently

- particles are in a quantum harmonic potential around their equilibrium position
- all atoms vibrate with frequency ω



[Source: *Hyperphysics*]

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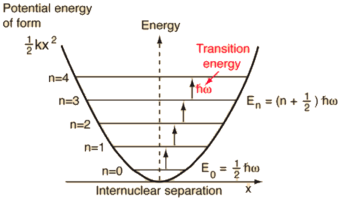
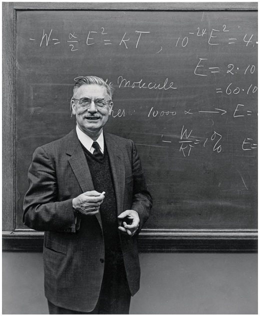


[Source: *Hyperphysics*]

This model was only partly successful:

- **Success!** – explained the **Dulong-Petit Law** (specific heat → constant at high temperature)
- **Failure!** – could not explain the T^3-dependence of specific heat at low temperatures.

Lattice Vibrations: Debye Model



Peter Debye [Source: *pubs.acs.org*]

Peter Debye instead proposed that the atoms were connected by springs so that the crystal was a coupled oscillator.

- atoms do not vibrate independently
- frequencies are not equal (and not equally common)
- N atoms in 3 dimensions means $3N$ normal modes

Lattice Vibrations: Normal modes?

What are normal modes again?

Lattice Vibrations: Normal modes?

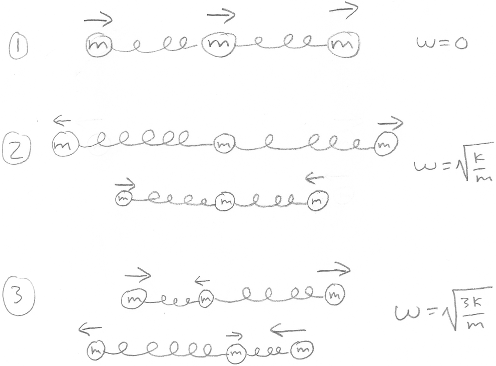
What are normal modes again?

Suppose we have three atoms (of mass m) connected by springs (of spring constant k) vibrating in one dimension:

Lattice Vibrations: Normal modes?

What are normal modes again?

We then will have 3 normal modes:

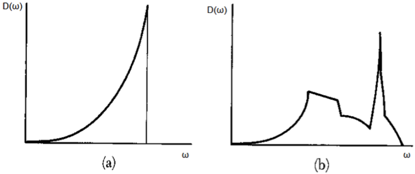


Lattice Vibrations: Debye Model

For the atoms of the Debye model:

- N atoms in 3 dimensions means $3N$ normal modes
- the number of modes increases as ω^2
- there is a maximum frequency – the **Debye frequency** – ω_D (because there are only $3N$ modes)
- we can also define the **Debye temperature**, Θ_D:

$$\hbar\omega_D = k_B\Theta_D$$



(a) Debye model density of states. (b) A realistic density of states.

[Source: Fig. 5-14, *Introduction to Solid State Physics, 3rd Ed.*, Kittel, 1966]

Lattice Vibrations: Phonons

Instead of picturing waves, we can think about *particles*.

The energy contained in a particular mode is given by the quantum harmonic oscillator energy,

$$E_n = (1/2 + n)\hbar\omega,$$

where ω is the frequency of that mode and n describes the quantum energy state.

A vibrational mode can only gain or lose energy in discrete amounts, and these quanta of heat energy are called **phonons**.

A mode in the nth energy state is occupied by n phonons, each with energy $E_p = \hbar\omega$.

Lattice Vibrations: Where do we use this?

We will use the Debye Model in:

- Electrical Resistivity
    - Low and high temperature resistivity in metals
- Specific Heat
    - Low and high temperature specific heat in metals
- Mössbauer Effect
    - Used to explain the origins of the effect

We will use phonons in:

- Optical Absorption Edge
    - "Indirect" absorption in semiconductors involving both a photon and a phonon

Superconductivity

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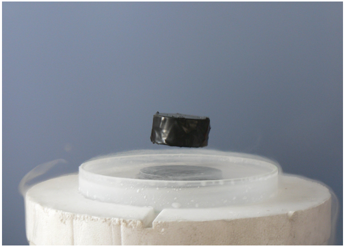
Superconductivity

Superconductivity: Introduction

At very low temperatures, some metals undergo a transition from normal conductor to **superconductor**

- ZERO resistivity
- expulsion of magnetic field lines (the **Meissner effect**)



[Source: *Wikimedia Commons*]

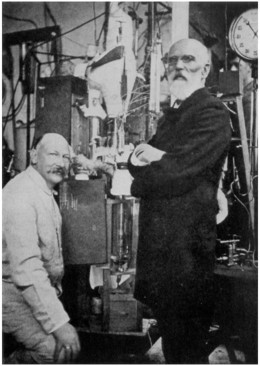
Typical superconducting temperatures are $T_C \leq 10K$.

Superconductivity: Where do we use this?

We will observe superconductivity in:

- Electrical Resistivity
  - Observe the drop to zero resistivity in niobium, vanadium and tantalum
- Specific Heat
  - Observe a discontinuity (and change in shape) in the specific heat of niobium
  - Measure the ratio of normal to superconducting specific heats

Good luck this quarter!



Heike Kamerlingh Onnes and Johannes van der Waals with the helium “liquefactor” in Leiden (1908)

[Source: *Wikimedia Commons*]