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Investigating thin film interference with a digital camera

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Thin film interference is discussed in most introductory physics courses as an intriguing example of wave interference. Although students may understand the interference mechanism that determines the colors of a film, they are likely to have difficulty understanding why soap bubbles and oil slicks have a distinctive set of colors—colors that are strikingly different from those present in the rainbow. This article describes a way to model these colors and a simple method for investigating them using a digital camera and a computer.
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I. INTRODUCTION

Materials that appear clear often give rise to distinctive colors, for example, the blue of the sky, the spectrum of a rainbow, the glow of a neon gas, and the swirls of color on the surface of a bubble. The blue of the sky is indicative of Rayleigh scattering, the rainbow suggests dispersion, the distinct spectrum from a gas suggests atomic emission, and the iridescent colors in a bubble are due to thin film interference. Although relating the colors of a blue sky, rainbow, and neon glow to the mechanisms that create them is often part of instruction, relating thin film interference to the colors from thin films is less straightforward.

Typical instruction of thin film interference introduces students to the constructive and destructive nature of wave superposition. White light reflecting off of a single interface reflects all wavelengths equally for nondispersive media. Adding a second film behind the first increases the intensity of certain reflected wavelengths while reducing and even eliminating the intensity of other wavelengths. Although no other mechanism is necessary to understand the colors of a thin film, this explanation is incomplete for predicting which colors will be observed.

In the following, we discuss a method that succinctly connects thin film theory and the colors of thin films. An assumption that simplifies this method models white light as composed of three representative wavelengths, those of the peak sensitivities of the cones of the eye: red, green, and blue. As we will show, this assumption is remarkably good in capturing the essential physics of thin films. This tricolor aspect of color vision is also employed by digital displays and cameras. The method may be employed in a classroom and requires only a digital camera, a computer, and some free and/or readily available software.

II. BACKGROUND

A standard introduction to thin film interference calls students' attention to light reflected from the two interfaces of a thin film, as happens with a soap bubble or an oil slick. Incident light reflects from a thin film with an inversion of its waveform at the first interface (because the transmitted wave moves from a faster to a slower medium). Reflection at the second interface occurs without an inversion of the wave (as the transmitted wave passes from a slower to a faster medium). The amplitude of the superposition of these reflected waves depends strongly on the phase difference δ of the second reflected wave relative to the first. For reflection at normal incidence, we have

$$\delta = \pi\left(\frac{2d_n}{\lambda} + 1\right), \tag{1}$$

where λ is the vacuum wavelength of light, $d_n = nd$ is the effective thickness of the film, n is the index of refraction of the reflecting media, and d is the film thickness. The amplitude of the reflected wave is

$$\mathcal{A} \propto \left|\cos\left(\frac{\delta}{2}\right)\right|. \tag{2}$$

The prefactor is unnecessary in the following discussion. The intensity $\mathcal{I}$ is given by the square of the amplitude,

$$\mathcal{I}(\delta) = |\mathcal{A}|^2 \propto \cos^2\left(\frac{\delta}{2}\right). \tag{3}$$

If the phase difference δ is an odd multiple of π, destructive interference occurs, and (taking both reflected waves to be the same amplitude) there is no reflected intensity. This cancellation of the reflected light occurs when

$$d_n = \tfrac{1}{2}m\lambda \quad (m = 0,1,2,\ldots). \tag{4}$$

Equation (4) is commonly presented in introductory courses and is employed in this setting to determine the wavelengths that are absent in a reflecting film of thickness d_n.

It is a much different question and one that more directly addresses our everyday experience to ask what color is observed when white light is incident on the film. In the following, we describe these colors and introduce an elementary method for observing and analyzing them that is suitable and accessible to introductory students. No other relations other than Eqs. (1)–(3) are necessary for this investigation.

III. THE COLORS OBSERVED FROM THIN FILMS

An effective way to determine and understand colors reflected from a thin film employs the trichromatic aspect of color vision. We can represent a given color by describing the degree to which the wavelength(s) of light that comprise

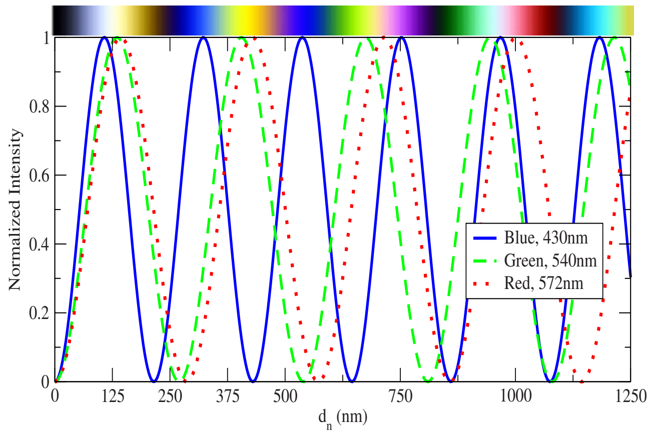

Fig. 1. (Color online) The intensity of reflection at normal incidence off a film of thickness d_n according to Eq. (3) for wavelengths corresponding to the peak sensitivities of the three RGB cones. A panel is shown at the top determined from these intensity profiles.

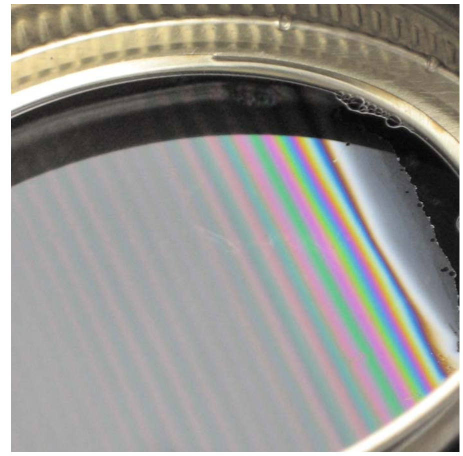

Fig. 2. (Color online) Image of a thin soap film in incandescent light. The film is tilted slightly so that it is thinner on the right of the image.

that color trigger a response in the three types of cones of the eye. The cones, responsible for color vision, respond to a range of wavelengths and each is most sensitive in unique ranges of the spectrum. The peak sensitivities are approximately 430 nm for the blue (or short) cone, 540 nm for the green (or medium) cone, and 572 nm for the red (or long) cone.[1] Electronic displays take advantage of this trichromatic aspect of color vision by varying the intensities of three colors (the subpixels red, green, and blue, or RGB). Such displays can create a nearly complete range of colors.

We can model white light as a superposition of these three distinct wavelengths and investigate what happens when white light is reflected off of a thin film. To fully describe the observed colors, the model must be generalized to better represent the spectrum of incident light and to account for the response of the cones to various wavelengths. This simplification provides a quick and intuitive approach to determine and interpret the colors of the films while still capturing the essential physics and predicting the progression of colors reflected from the film to a remarkable degree.

The intensity of the reflected light for the three wavelengths of peak sensitivity is shown in Fig. 1 as a function of the thickness of the film. The intensities are normalized to unity, and it is assumed that the reflected light from both surfaces of the thin film have the same amplitude. In Fig. 1 the observed color is determined by the sum of the plotted intensities, for example, equal intensities of red (R) and blue (B), but an absence of green (G), indicate magenta. Red and green light, absence of blue, create the perception of yellow. A band of colors determined from the intensities of Eq. (3) is depicted in Fig. 1. More on the primary colors of light and observed colors can be found in introductory texts on color.[2]

We consider first the color composition of very thin films in Fig. 1. Beginning on the far left, there is no reflection of visible wavelengths off very thin films ($d_n \lesssim 50$ nm) because they all destructively interfere. All R, G, and B wavelengths are transmitted, and the film appears clear. If we make the film thicker, the first wavelength not diminished by the second reflection from the film is blue, and a dark blue light is observed. For $d_n \approx 135$ nm, all of the wavelengths constructively interfere and are present with equal intensities and white light reflects off of the film.

Thickening the film beyond the first white band first attenuates the blue light and this combination of red and green light brings about the perception of yellow. In this region, where the blue intensity is minimal, the green light diminishes faster than the red, and the yellow reddens until the blue dominates again near $d_n \approx 270$ nm. This sequence of colors, which is predicted by the simplified, trichromatic model of the reflected light, describes the sequence of colors shown in the thinner portions on the right of the photograph in Fig. 2.

In the range of film thicknesses greater than the first minimum ($d_n \approx 0$ nm), where all visible wavelengths destructively interfere, and less than the maximum ($d_n \approx 135$ nm), where all visible wavelengths constructive interfere, each color reaches a minimum separately. Blue is a minimum at $d_n = 215$ nm, then green at $d_n = 270$ nm, and red at $d_n = 286$ nm. One primary color of light is removed sequentially, and, though not exact, the colors present are similar to those of the primary colors of pigments: Cyan (blue-green), followed by yellow (green-red), and then magenta (red-blue). The sequence of colors for these thicknesses, $d_n \approx 215-430$ nm, is shown in Fig. 2. These colors are composed of multiple wavelengths and are present due to the subtractive mechanism of the film. They are not the monochromatic colors of the rainbow, which explains succinctly the difference in character of the colors from a thin film to that of a rainbow.

Newton[3] first described the progression of colors of thin films ("Newton's colors"), which he observed in the rings made by pressing two convex lenses together. He not only observed the colors present when white light is incident on a film, but used a prism to refract the incident light. By observing the resulting bands of colors using this refracted light as the incident beam, Newton concluded that the colors from a thin film illuminated with white light were not "homogeneal" (monochromatic), but "heterogeneal" or "compound."[3]

IV. AN RGB ANALYSIS OF COLORS

Exploring Newton's colors by examining their red, green, and blue components is simple with a digital camera because each pixel records the RGB intensity. A digital image of a soap film is shown in Fig. 2. The film was created with dish soap and water on a canning jar lid.[4] The incident light is

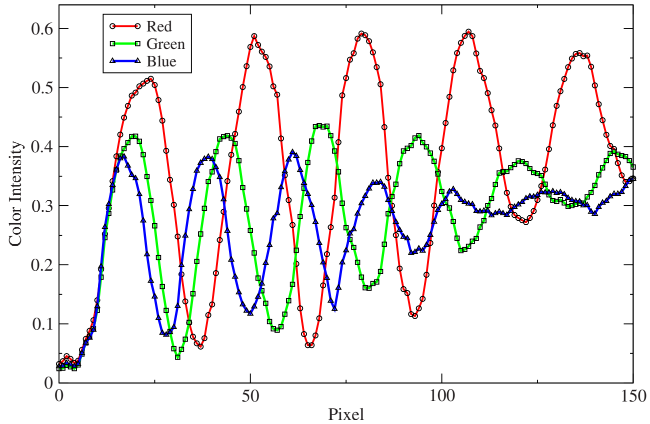

Fig. 3. The observed subpixel intensity for a column of pixels perpendicular to the band of colors generated by reflection off a soap film in incandescent light. The film thickens with increasing pixel number. The intensity data are generated by MATHEMATICA using a jpeg image from a digital camera.

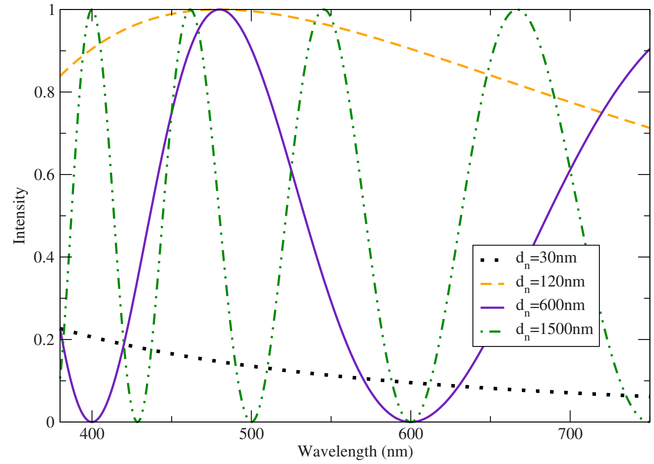

Fig. 4. The intensity response of visible wavelengths for reflection at normal incidence off films of varying thickness according to Eq. (3).

from a tungsten bulb diffusely reflected from a white wall, and a black backdrop (a black plastic cup) absorbs transmitted light. The film (liquid dish soap and water) is tilted with respect to the floor so that bands of color appear as the film gradually thins. A thinner film is created at the top and a thicker film at the bottom. Creating and photographing films is easy to do by students and is instructive because it calls attention to the transmitted light[5] and the angles necessary to see the specular reflection. Also students can see the gradual thinning of the bubble and the emergence of various colors. Our photos are available for analysis.[6]

Figure 5(a) shows a section extracted from one of these images. Note that the predicted progression of colors for very thin films is black, dim blue, white, yellow, and red, as predicted. It is simple to examine the color makeup, that is, RGB content, of pixels along a line perpendicular to the film's bands of color, with software that can determine the RGB content of a pixel.[7]

We could perform the analysis by examining the color pixel-by-pixel. Although such an approach is tedious, it does familiarize students with some principles of light mixing. For example, many students are surprised that red and green subpixels give the appearance of yellow when lighted simultaneously. A more direct method of analyzing the color makeup of these pixels is possible. An image may be imported into (for example) MATHEMATICA, which can quickly convert a single column of pixels into arrays of RGB content values. Minimal technical skills are required and a complete intensity pattern, as shown in Fig. 3, is created. (The incandescent bulb, which has a peak intensity in the infrared, is responsible for the relative prominence of red wavelengths.) Although different image processing formats will yield different absolute intensities, the relative intensities of the RGB values and periodicity of fringes will remain the same.

Immediately apparent from the RGB intensities in Fig. 3 are the periodic peaks of constructive interference, followed by destructive interference. The interference fringes clearly indicate that blue has the shortest wavelength because it destructively interferes first, followed by green, and then red. Consistent with the predictions, the thin film shifts from black (no pixels lit, or lit only weakly) to white (all pixels brightly lit), and then to red. From these values, we can investigate the thickness of the film. (Note that some of the perceived change in thickness is due to the viewing angle and not the absolute thickness of the film; that is, even if the thickness of the film was uniform, we would see slight variations in color due to the angle of the reflected light passing through the film. The variation in color due to the viewing angle gives rise to the iridescent characteristic of thin film interference. For our setup, viewing a 5 cm film from a distance of 50 cm, this angular effect is negligible.) If the index of refraction for the soap film is known and approximations for red, green, and blue wavelengths are made (such as those given in Sec. III), then approximating the absolute thickness of the film is straightforward using Eq. (4).

V. LIMITATIONS

One limitation that should be addressed is the validity of the three wavelength approximation. We have used the response of three wavelengths, 430, 540, and 572 nm, to be representative of the full, integrated intensity response for the range of wavelengths perceived by the short, medium, and long cones of the eye, respectively. Despite its very reasonable predictive abilities for extremely thin films, it is natural to consider the limitations of this model.

Figure 4 illustrates the intensity response for the range of visible wavelengths for a variety of film thicknesses. For an extremely thin film, the intensity does not vary considerably with wavelength, and the response of a single wavelength is representative of the intensity response for the neighboring wavelengths. As the film thickens, the intensity begins to vary rapidly with wavelength. For a wavelength interval (for example, one characteristic of the cones of the eye) reflecting from thicker films, there are multiple intensity minima and maxima, as seen for $d_n = 1500$ nm in Fig. 4.

When we consider a range of wavelengths, the minima and maxima will occur for slightly different thicknesses than predicted by the one-wavelength approximation of Eq. (3). In addition, the maxima and minima are neither as bright nor as dim, as predicted by Eq. (3), due to the multiple constructive and destructive wavelengths within the range of colors perceived by a given cone. This "washing out" of the maxima and minima is particularly true for thick films, where the intensity response varies rapidly with wavelength, and for shorter wavelengths, which are more sensitive to changes in thickness, as seen for $d_n = 1500$ nm in Fig. 4.

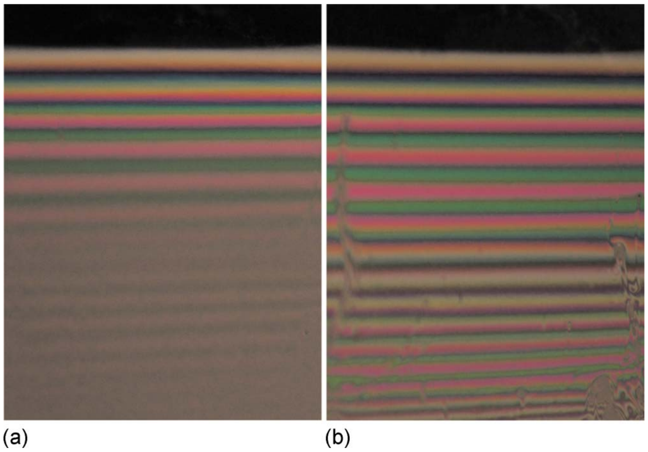

Fig. 5. (Color online) A thin film in (a) incandescent light and (b) compact fluorescent light. Note the washing out of colors in the film under incandescent light compared to the same film in fluorescent light.

This "wash out" is apparent in Fig. 3, where the blue subpixels show little variation in intensity with increasing thickness for thick films. The presence of this relatively unmodulated blue results in the perception of magenta (blue+red) and cyan (blue+green) bands of color in thicker films, as seen in Fig. 2. For very thick films, the green and red wavelengths also diminish to relatively uniform intensities, and as a result, the films appear uniformly gray because all three primary colors are present. Once the film has thickened to appear gray, the interference patterns persist but they are neither visually apparent nor detectable by a digital camera because they have been "washed out" by neighboring wavelengths. Observing interference patterns from a white light in this region requires a much finer resolution of wavelengths than the camera or the eye provides. Further details regarding these conclusions can be found in the Appendix.

VI. EXTENDING THE INVESTIGATIONS: FLUORESCENT SOURCES

One common extension that enables an investigation of interference patterns in thicker films is to use a monochromatic source, which prevents the washing out of interference fringes caused by neighboring wavelengths. This approach will not display the iridescent multiple-wavelength colors. Alternatively, a fluorescent "white" light can be used because the distinct, compact peaks in its spectra, created from a variety of fluorescing phosphors, prevents the dimming to gray characteristic of the more uniform blackbody spectrum of the tungsten bulb, as shown in Fig. 5(b). Observing a thin film with an incandescent source and then switching to a fluorescent white light source produces a dramatic shift in the observed colors, illuminating the fringe patterns in thicker films that appear gray under incandescent lights.

If we use the methods in Sec. IV using a compact fluorescent light as a source (a 14 W Commercial Electric brand, model EDX0-14), we find the RGB content for a line of pixels perpendicular to the bands of color, as shown in Fig. 6. (Note that because of different resolutions of the images as well as different films, these pixels cannot be directly compared to those from Fig. 3.)

Of these intensity profiles, blue offers a particularly interesting pattern of fringes due to two strong peaks in the blue

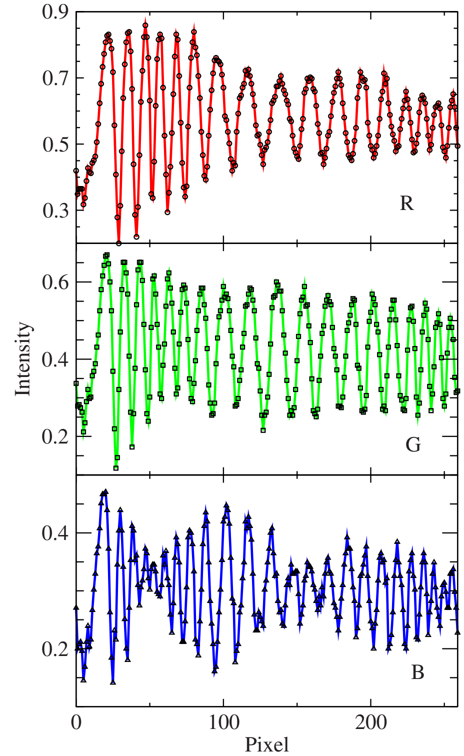

Fig. 6. The observed subpixel intensity for a column of pixels perpendicular to the band of colors generated by reflection off a soap film in fluorescent light. Data are generated by MATHEMATICA using a jpeg image from a digital camera, as in Fig. 3. The three colors are separated for ease of viewing. Note the beat pattern, most apparent in the response of blue wavelengths.

part of the spectrum. The strongest peaks at 436 and 488 nm are both perceived by the blue cone in the eye, and their proximity allows us to see a beat pattern in the fringes. Although these two frequencies are the strongest peaks in the blue part of the spectrum, there are a range of additional frequencies present. In particular, a broader, less intense blue peak at 450 nm causes the fringe pattern to diminish as the thickness increases.

VII. DISCUSSION

Understanding colors reflected from thin films is a rich area of exploration for introductory courses. It requires an elementary understanding of both the causes of thin film interference and the biological basis of color vision. It offers a strong match between theory and experiment. Extending the investigation to consider fluorescent light brings in considerations of atomic sources of light. It also touches on constructive and destructive interference, beat patterns, and other wavelike phenomena.

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clusions, or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

APPENDIX: EXTENDING THE THREE WAVELENGTH APPROXIMATION

We consider the average intensity perceived by one RGB cone in the eye, which is an average over the interval of spectral sensitivity for that cone. Incorporating a full interval of wavelengths reflected from a thin film enables a few insights into the colors observed from thicker films.

Two assumptions are made in the following to simplify the calculation. We consider only a uniform spectrum and step-function absorption; that is, each frequency is present with the same intensity and all wavelengths are absorbed equally. Although each cone perceives a broad range of frequencies, we will consider only the range of frequencies that are strongly perceived. This strongly perceived range of wavelengths is narrow, less than 100 nm,[1] and comparable for the short, medium, and long cones.

The normalized average value of the intensity reflected from a film of thickness d_n for a range of wavelengths is given by

$$\bar{\mathcal{I}} = \frac{1}{(\lambda_2-\lambda_1)}\int_{\lambda_1}^{\lambda_2}\cos^2\frac{\pi d_n}{\lambda}d\lambda = \frac{1}{(\lambda_2-\lambda_1)}\left(\lambda\cos^2\frac{\pi d_n}{\lambda}+\pi d_n \mathrm{Si}\left(\frac{2\pi d_n}{\lambda}\right)\right)\Bigg|_{\lambda_1}^{\lambda_2}, \tag{A1}$$

where λ_1 is the lower threshold, λ_2 is the upper threshold, and $\mathrm{Si}(x)$ is the sine integral. It can be expanded for large arguments, that is, thick films compared to the wavelength, to yield

$$\pi d_n \mathrm{Si}\frac{2\pi d_n}{\lambda} = \frac{\pi^2 d_n}{2} - \sin\frac{2\pi d_n}{\lambda}\left(\frac{\lambda^2}{4\pi d_n} - \ldots\right) - \cos\frac{2\pi d_n}{\lambda}\left(\frac{\lambda}{2}+\ldots\right). \tag{A2}$$

If the range of wavelengths is small relative to the wavelength itself, Eq. (A1) simplifies to

$$\bar{\mathcal{I}} \approx \frac{1}{2} - \frac{(\lambda_1+\lambda_2)^2}{8\pi d_n(\lambda_2-\lambda_1)}\sin\frac{\pi d_n(\lambda_2-\lambda_1)}{\lambda_2\lambda_1}\cos\frac{\pi d_n(\lambda_2+\lambda_1)}{\lambda_2\lambda_1}. \tag{A3}$$

This intensity can be compared to that in Eq. (3), which is shown in Fig. 1. Several new features are present here: There is an envelope that decays to a limiting average intensity, the intensity profile has a beat pattern, and minima and maxima fall at different thicknesses d_n than those of Eq. (3), all due to the influence of the finite wavelength interval.

As noted, the second term of Eq. (A3) diminishes as d_n^{-1} so that the average normalized intensity goes to 1/2 for very thick films. In these thicker films, the intensity varies so rapidly across a given range of wavelengths that the modulation of the integrated intensity is negligible for that wavelength interval. For these large thicknesses, and the three RGB wavelength intervals all contributing, the observed color is uniform gray.

The second term of Eq. (A3) represents a beat pattern with decreasing amplitude. The d_n^{-1} prefactor decreases the maximum intensity with increasing thickness due to a washing out of constructively interfering wavelengths by neighboring destructively interfering wavelengths (see Fig. 4). The second factor in the second term of Eq. (A3) (the sine term) represents a beat pattern with a frequency of

$$f_{\mathrm{beat}} = \frac{\lambda_2-\lambda_1}{\lambda_1\lambda_2}. \tag{A4}$$

The beat minimum, the point at which this second term is zero, occurs for thicknesses where the number of completely constructively interfering wavelengths equals the number of completely destructively interfering wavelengths in the interval $\lambda \in [\lambda_1,\lambda_2]$. The cosine term in the modulating portion of Eq. (A3) represents a modulating frequency and is a characteristic of the oscillating intensity response of a single wavelength with frequency,

$$f_{\mathrm{mod}} = \frac{\lambda_2+\lambda_1}{2\lambda_1\lambda_2}. \tag{A5}$$

This frequency, Eq. (A5), is approximately the frequency of the intensity response of the average wavelength for interval $[\lambda_1,\lambda_2]$ and is consistent with Eqs. (3) and (4) in the limit λ_2 to λ_1.

The envelope in Eq. (A3) allows us to investigate how rapidly each color will dim or wash out. The maximum intensity before the second beat minimum is unity. After this maximum, the maximum intensity is significantly diminished. As an estimate, we evaluate the sine portion of the envelope in Eq. (A3) at the second maximum and find that the range in intensities is approximately

$$\frac{2(\lambda_1+\lambda_2)^2}{12\pi\lambda_2\lambda_1} \approx \frac{2}{3\pi}\left(1+\frac{(\lambda_2-\lambda_1)^2}{4\lambda_1^2}+\ldots\right) \tag{A6}$$

for a small range in wavelengths. That is, after the first maximum, the reflected intensity oscillates, but the intensity range is only about 21% of the initial intensity range. The diminishing intensity and washing out to gray can be seen in the film in Figs. 2 and 5(a).

We can therefore regard a color to have washed out when the film thickness is such that the color has reached the second minimum, that is, when

$$d_n \gtrsim \frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1} = f_{\mathrm{beat}}^{-1}. \tag{A7}$$

As the film thickness increases, the range of intensities of the fringes is diminished (the fringes are neither as dark nor as bright), and the result is an increasingly uniform medium-intensity color with increasing thickness. Blue, with its shorter wavelengths, is the first to dampen its oscillations because it washes out first according to Eq. (A7). At these thicknesses, red and green continue to oscillate in intensity with increasing film thickness, which results in the familiar magenta and cyan color bands for these thicker films, as in Figs. 2 and 5. Blue wavelengths are also nearly washed out in Fig. 3.

For even greater thicknesses d_n, those exceeding the threshold given by Eq. (A7) for washing out both the green and red oscillations, all three effective RGB intensities approach the average value given in Eq. (A3) and the film appears gray.

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[1]See, for example, L. T. Sharpe, A. Stockman, H. Jagle, and J. Nathans, in *Color Vision: From Genes to Perception*, edited by K. R. Gegenfurtner and L. T. Sharpe (Cambridge U. P., Cambridge, 1999), pp. 3–52.

[2]See, for example, D. R. Falk, D. R. Brill, and D. G. Stork, *Seeing the Light: Optics in Nature, Photography, Color, Vision, and Holography* (Harper and Row, New York, 1986), pp. 238–265.

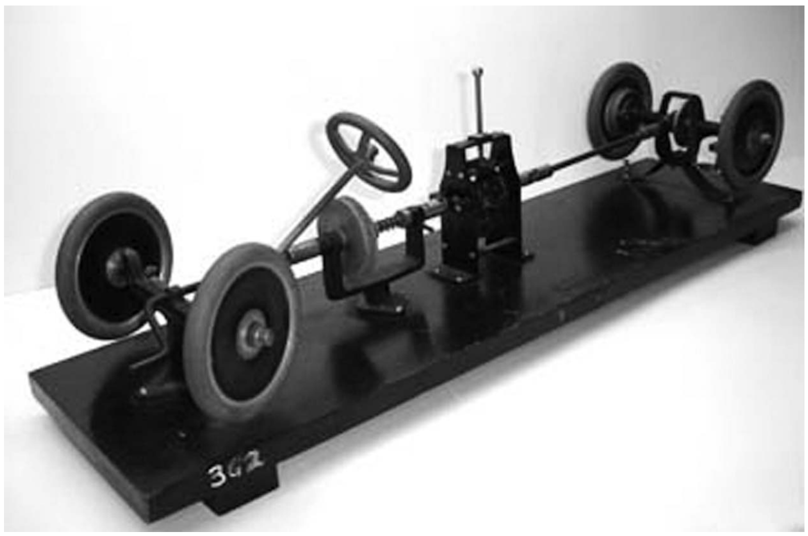
[3]I. Newton, *Opticks*, 2nd ed. (Dover, New York, 1952), Pt. I.

[4]We used Trader Joe's brand dish soap. Some online references suggest using glycerine to extend the life of the thin film. We found that glycerine did prolong the life of the film but greatly affected the thickness of the film; it no longer thinned uniformly across the film but swirls of color were often present.

[5]R. Newburgh and D. Goodale, "Student difficulties in analyzing thin-film interference," Phys. Teach. **47**, 227–230 (2009).

[6]Our photos are freely available from ⟨phys.csuchico.edu/latkins/films⟩. The photos were taken using a Canon Rebel ti1 on a low-resolution setting. We found that a high resolution is neither necessary nor useful.

[7]See, for example, Adobe Illustrator, IDL, and free software such as IMAGEJ.



Automobile Chassis Model. In an earlier segment in this series, the four cylinder automobile engine model in the 1928 Welch catalogue was pictured. The same catalogue shows the Automobile Chassis Model, priced at $100. The individual parts, including the steering gear, clutch, gear shift, universal joint and differential and brakes, could also be bought separately. The individual gear shift model was driven by an electric motor, and cost $27.50. This model, mounted on a base 75 cm long, is in the collection of Michael Payne, who also supplied the photograph. (Notes by Thomas B. Greenslade, Jr., Kenyon College)